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Student's T-Test

Student's T-Test tests whether the average (**mean**) of two populations are the same.

> Hypothesis: These two populations are different because they have different averages.
> Null Hypothesis: These two populations are not different because they have the same averages.

Instead of eyeballing the **means** and saying "They look different to me" a t-test evaluates the difference by looking at the **means** in light of how far the points within each population differ from the **mean**: the **variance**. Basically, a t-test is the difference in **means** divided by differences in **variance**. The populations are different if t is large. The more the **means** differ, the larger t will be. The closer points within a population are to the **mean** (the smaller the **variance**), the larger t will be. A large t suggests that if you drew bell-shaped curves of these populations, you would find very little overlap. How much overlap can you have and still say the populations are different? How large must t be before you can say, "I have two different populations?" That's where the **probability value** (**p**) comes in.

To do a t-test, you must

1) know the **sample size** of each population (**n**),
2) calculate the **mean** of each population (**x**),
3) calculate the **variance** of each population (**s^2**),
4) calculate t,
5) decide the confidence you want to have that the populations are different (**p**), and
6) use a table to see if t exceeds the **probability value** (**p**) that expresses your confidence.

After you have done all this, if t is large, you will be able to say, "I reject the null hypothesis that these populations are the same. I am 95% confident they are different because with my sample size and with my value of t, the probability of being wrong is less than 0.05." If t is small, you will have to say, "Because of my sample size and because my value of t is small, I cannot say the two populations are different and I cannot reject the null hypothesis." In which case you give up, or go out and collect more data to get a larger sample size, or you design another experiment and try again.

To calculate a t-test, you need to understand **mean** (**x**), **variance** (**s^2**), and **normal distribution**. To evaluate the results, you need to understand the **probability value** (**p**) and **degrees of freedom** (**d.f.**).

Mean (**x**) is the average of each population. If you have a **sample size** (**n**) of 10 leaves and you measure the area of each leaf (**a**), if you add up all the areas and divide by 10 and you will get the **mean** area for these 10 leaves.

$$\textbf{Mean} = X = \frac{\sum a}{n}$$

Variance (**s^2**) is the spread of points within the sample about the **mean**. When you throw darts at a dartboard, **variance** is how far away from the bulls-eye (the **mean**) your darts fall: the further from the bulls-eye, the greater the **variance.** Assuming you don't improve with practice, the more darts you throw the closer your **variance** will approach the **mean.** This is why **sample size** (**n**) is important. The implication is that the more you measure, the closer the calculated **mean** of your sample is going to be to the true **mean** of the entire population.

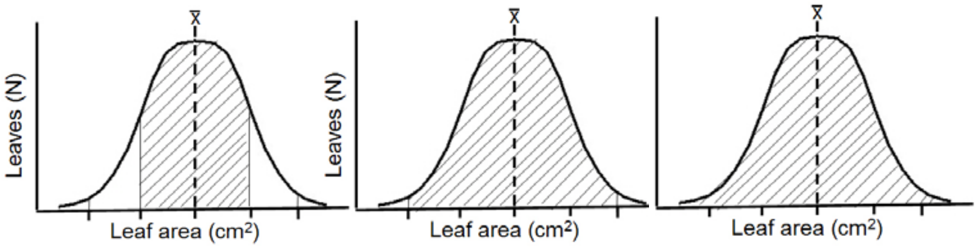
$$\textbf{Variance} = s^2 = \frac{\sum (X - a)^2}{n - 1}$$

Variance (**s^2**) can be used to calculate **Standard Deviation** (**s**).

$$\textbf{Standard Deviation} = s = \sqrt{s^2}$$

Standard Deviation (**s**) becomes important for estimating the distribution of your population. In normally distributed populations, one **standard deviation** away from the **mean** (**$x \pm s$**) includes 2/3 or 68.3% of the population: **$x \pm 2s = 95.4\%$**, and **$x \pm 3s = 99.7\%$**. **Standard deviations** should help you create a mental image of the bell-shaped curve of the population.

$x \pm s$ (68.4%) **$x \pm 2s$** (95.4%) **$x \pm 3s$** (99.7%)



The **t-test**: to conduct a **t-test**, you need to calculate the **means** of both populations (**x_1** and **x_2**), the **variance** of both populations (**s_1^2** and **s_2^2**) and know the **sample size** of each population (**n_1** and **n_2**). Note: you need the absolute value of the difference in **means**, and you take the square root of **$(s_1^2/n_1 + s_2^2/n_2)$**.

$$t = \frac{|x_1 - x_2|}{\sqrt{(s_1^2/n_1 + s_2^2/n_2)}}$$

Probability value (**p**): for each value of **t**, there is a value (**p**) which indicates the probability that the null hypothesis is true and the two populations are the same, however different they may look. The tradition in Biology is to reject the null hypothesis when $p<0.05$. At this probability value, you will reject the null hypothesis when it really is true 1 time out of 20, or less than 5% of the time. To find out whether **t** is significant and whether you can reject the null hypothesis, you must first determine the acceptable **p-value** and determine your **degrees of freedom**.

$$\textbf{Degrees of Freedom} = d.f. = n_1 + n_2 - 2$$

Can you reject the null hypothesis? Use your calculated **degrees of freedom** and your **p-value** to find **t** in the table below. If your value of **t** is greater than the value listed in the table, then you can reject the null hypothesis.

Critical Values of Student's t

Probability

d.f.	0.20	0.10	0.05	0.01
1	3.078	6.314	12.707	63.657
2	1.886	2.920	4.303	9.925
4	1.533	2.132	2.776	4.604
10	1.372	1.812	2.228	3.169
20	1.325	1.725	2.086	2.845
30	1.310	1.697	2.042	2.750
60	1.296	1.971	2.000	2.660
120	1.289	1.658	1.980	2.617

When you report your results, you need to give **t**, **d.f.**, and **p**.